



Optimizing Ventilation Coefficients for High Speed Underwater Vehicles

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Abstract

The innovative approach of ventilated supercavitation offers a pathway to reduce drag around underwater vehicles by creating a gas-filled cavity. The formation ventilation coefficient is pivotal in determining the gas volume needed for supercavity creation. Traditional empirical models for predicting this coefficient are based on free surface water experiments, which do not reflect the conditions in closed-wall water tunnels where wall effects can alter supercavity dynamics.

This study introduces a new empirical model that leverages existing experimental data to accurately predict the ventilation needs for supercavities in such tunnels, taking into account the blockage effects of tunnel walls. The model's validity extends across a wide range of Froude numbers and has been corroborated with various experimental data sets. Utilizing a pattern search optimization algorithm, the model's empirical constants were fine-tuned, showcasing its capability to reliably predict the relationship between the ventilation coefficient and cavitation number across different tunnel environments. It also accurately determines the necessary ventilation rate for supercavity initiation and maintenance. In essence, this research provides a refined tool for optimizing underwater vehicle performance, contributing to the advancement of marine vehicle engineering by enhancing efficiency through precise ventilation control.

Keywords: Ventilation rate, artificial supercavitation, closed-wall water tunnel, pattern search optimization, supercavity formation

Introduction

When the pressure reaches the vapor pressure of a liquid in specified regions of fluid flow, cavitation will take place. If the cavitation bubble is so large that surrounds the object which forms it, this special case of cavitation is named supercavitation [1]. One of the most important applications of the supercavitation phenomena is to reduce the friction drag of an object which travels underwater [2]. This dramatic decrease in skin friction drag allows the vehicle which travels underwater to achieve very high speeds above 100 m/s [3]. Supercavitation can be obtained naturally or artificially. In the case of natural supercavitation, the pressure reduction in the liquid naturally occurs due to the fast movement of the object in the liquid [4]. However, the air or a non-condensable gas is injected around the object for the formation of an artificial or ventilated supercavity [5]. For underwater vehicles, the cavity is created by a cavitator at the vehicle nose which is usually a disk or some other shapes such as cone.

Cavitation number is a dimensionless number which specifies the extent of the cavitation and is expressed as [6]:

$$\sigma = \frac{P_{\infty} - P_c}{0.5\rho V_{\infty}^2} \quad (1)$$

where P_{∞} and P_c are the ambient pressure and the cavity inside pressure respectively. ρ is the fluid density and V_{∞} is the mainstream velocity. It should be noted that supercavities are often attained at cavitation numbers less than 0.1 [7].

Froude number is another important dimensionless number in studying supercavitation which defines as follows [8]:

$$Fr = \frac{V_{\infty}}{\sqrt{gD_n}} \quad (2)$$

where D_n is the cavitator diameter and g is the gravitational acceleration. The value of the Froude number shows the ratio of the inertial force to the gravitational force and determines the effect of gravity on the shape of the cavity and its deflection from the straight line. As a ventilated supercavity can be obtained at relatively lower speeds than the natural case for the same cavitation number, the effect of the Froude number is significant for ventilated supercavities [9].

As mentioned, the air or a non-condensable gas is injected into the cavity for the formation of an artificial or ventilated supercavity. The ventilation coefficient is defined as follows, where \dot{Q} is the volumetric flow rate of the injected gas into the cavity [10].

$$C_Q = \frac{\dot{Q}}{V_{\infty} D_n^2} \quad (3)$$

Early studies in the field of supercavitation were focusing on the natural supercavitation and consist of many experiments and different empirical models of supercavities [11]. These early experimental studies were performed in free jet flow facilities and hence the attained cavitation numbers were very low values due to the absence of blockage effect of the water tunnel. The blockage is defined as the ratio of tunnel diameter to the cavitator diameter and determines the effect of tunnel walls on the inside flow. Semenenko [13] performed experimental studies on the dependency of ventilation coefficient C_Q to the cavitation and Froude numbers. The results showed that the ventilation coefficient versus cavitation number graph for each Froude number consists of three parts which are related to different mechanisms of gas leakage from the supercavity. The variations of ventilation coefficient vs. cavitation number can be described as follow. Values of $\sigma Fr > 1$ which correspond to the right part of the graphs are related to the first type of gas leakage namely the portion gas leakage or toroidal vortices. In this case, the formed bubbles of gas are detached from the tail of supercavity and convected downstream of the flow. The effect of gravity is negligible for the first type of gas leakage and therefore the detached bubbles of gas move downstream with a small deviation from the horizon. Also, values of $\sigma Fr < 1$ correspond to the second type of gas leakage, namely the vortex tubes or twin vortices. This type of gas leakage corresponds to the left part of the ventilation coefficient versus the cavitation number graph. The experimental results indicate that after reaching this region, the cavitation number doesn't change significantly by increasing the ventilation rate. The two mentioned mechanisms of gas leakage from ventilated supercavities are both stables. Both mechanisms of gas leakage can occur for the intermediate values of $\sigma Fr \approx 1$ and this transient regime of gas leakage is very unstable. This regime is related to the mid parts of the aforementioned graph. The criterion of transition from the first type of gas leakage to the second type which is called the Campbell-Hilborne criterion is expressed as follows [13]:

$$\sigma Fr \approx 1 \quad \text{for} \quad Fr \approx 5 - 25 \quad (4)$$

It should be noted that the conducted experiments by Campbell and Hilborne were at a free surface facility without the effect of blockage and the above equation will be different for experiments in water tunnels.

Cavitation water tunnels have been recently used in order to investigate the ventilated supercavity. Saint Anthony Falls Laboratory (SAFL) of the University of Minnesota has a cavitation tunnel which is very effective for cavitation and gas ventilation experiments and several experiments have been conducted at this laboratory. This water tunnel which is a closed recirculating tunnel has a test section of 120 cm (Length) \times 19 cm (Width) \times 19 cm



(Height) and is designed to operate at a maximum velocity of 20 m/s [1]. For example, Kawakami and Arndt [4] studied the artificial supercavitation behind a sharp-edged disk for various model configurations and presented the results of the supercavity shape, closure mechanism and the ventilation requirement for different Froude numbers. The experiments were conducted in the SAFL water tunnel and therefore the blockage effects were considered in the results. Karn et al. [14] conducted experiments in the SAFL water tunnel and studied the ventilated supercavity in unsteady flow. The unsteady flow was generated using a gust generator installed upstream of the model. They showed that the unsteadiness in the incoming flow can affect the cavity shape and also it can change the supercavity closure mechanism. Karn and Rosiejka [1] investigated artificial supercavities and studied the cavity appearance, closure mechanism and also investigated the air entrainment for the formation and sustenance of supercavities.

Also, the Chungnam National University (CNU) has a similar water tunnel to SAFL which is capable of ventilation experiments. The test section of this tunnel is smaller than the SAFL water tunnel and is 120 cm (Length) × 10 cm (Width) × 10 cm (Height), but the maximum velocity in this water tunnel is identical to the SAFL tunnel and is equal to 20 m/s. Ahn et al. [9] performed experimental studies on the same models at two different water tunnels of CNU and SAFL and compared the results with each other. They used the same scale models to consider the blockage effects of two water tunnels and showed that the results of both cavitation tunnels are in good agreement. They also measured the cavity inside pressure at different injected rates of gas and showed that the pressures measured at different positions inside the cavity are almost identical when a clear supercavity is maintained. Furthermore, they explored the effect of air injection location and direction.

Besides the experimental studies in the water tunnels, some numerical investigations are also presented which model the supercavitation. Wang et al. [15] performed experimental and numerical studies to investigate the artificial cavitating flow structure. Their numerical simulation was performed by CFX with a free surface model and a filter-based model. They observed the first and second mechanisms of gas leakage at different Froude numbers when the gas injection rate was constant. In another numerical study, Yang et al. [16] simulated the flow field around a three-dimensional body in supercavitating flow. Their numerical method was based on the multiphase computational fluid dynamics (CFD) combined with turbulence and cavity models. The presented results by Yang et al. [16] consisted of the variations of the supercavity diameter and cavitation number with entrainment rate, the variation of drag coefficient with cavitation number and the effect of gravity on the deformation of supercavity axis. They also compared their numerical results with experimental data of water tunnel and towing tank and showed that the obtained numerical results are in good agreement with experimental results. Also, there are some other numerical studies for simulation of supercavitation which can be found in the literature [17]. However, supercavitation is a complex and multiphase phenomenon and numerical studies have many difficulties to perform [9] and also need several experimental data to be validated.

The scope of the present paper is to develop an empirical-based model for ventilation coefficient in order to predict the proper ventilation rate of supercavities in the water tunnels before the formation and at the formation point of the supercavity by considering blockage wall effect. The value of the formation ventilation coefficient is an important parameter that determines the ventilation demand for the formation of ventilated supercavities. The proposed model is then validated by other available experimental data at a relatively wide range of Froude numbers. The empirical constants of the new proposed model are obtained using the pattern search optimization algorithm. It is shown that the model can predict the variation of the ventilation coefficient before the formation and the required ventilation rate for the formation of the supercavity with acceptable accuracy.

The rest of the paper is organized as follows: the method of developing the proposed empirical model is presented in the next section and then the results are presented and discussed in the third section. Finally, the conclusions are drawn.

Proposed empirical model

In this section, first, the governing equation of the ventilated supercavities will be presented. Then, the accuracy of two models which are the Spurk and Epshtein models will be evaluated for calculation of the ventilation rate of supercavities in water tunnels. Finally, the new empirical-based model will be presented.

The mass balance for the supercavity is written as:

$$\frac{d}{dt}(\rho_g Q_c) = \dot{m}_{in} - \dot{m}_{out} \quad (5)$$

where ρ_g is the density of injected gas to the supercavity, Q_c is the volume of supercavity, \dot{m}_{in} and \dot{m}_{out} are the mass flow rates of the gas injected into and escaping from the supercavity respectively. Using the Bernoulli equation and assuming that the gas is isothermal, the mass balance equation can be rewritten approximately as Eq. (6).

$$\frac{d}{dt}(P_c Q_c) = P_c \dot{Q}_{in} - P_{\infty} \dot{Q}_{out} \quad (6)$$

where \dot{Q}_{in} and \dot{Q}_{out} are the volume flow rates of the gas injected to and escaping from the supercavity respectively. For the steady supercavity, one can write the above equation as: $P_c \dot{Q}_{in} = P_\infty \dot{Q}_{out}$.

Fig.1 shows the experimental results of the ventilation coefficient v.s. cavitation number as the injection rate incrementally increases (C_Q increases from 0.02 to 0.16) and then decreases (C_Q decreases from 0.16 to 0.02) for a period of time. Experimental results show that as the ventilation rate of the gas to the cavity increases, the cavitation number decreases and reaches a minimum value [9]. The clear supercavity appears at this minimum cavitation number which maintains the cavity shape even if the ventilation rate decreased. This means that the amount of ventilation rate to sustain a supercavity is less than the amount of ventilation rate to create it [18]. It should be noted that the clear supercavity at minimum cavitation number is mainly a twin vortex type while before reaching the clear supercavity, the toroidal cavities or portion gas leakage occur. Therefore, based on Fig. 1 it can be supposed that by increasing the ventilation coefficient from 0.02 to 0.1 and before reaching the minimum cavitation number (approximately equal to 0.34) the mechanism of gas leakage is the portion gas leakage and by increasing the ventilation coefficient to 0.12 and reaching to the minimum cavitation number (0.34) the mechanism of gas leakage will change to the twin vortex mechanism. Therefore, the clear supercavity forms at ventilation coefficient equal to 0.12 which is named the formation ventilation coefficient or C_{Qf} . As can be seen in Fig.1, the twin vortex mechanism will continue by increasing the ventilation coefficient from 0.12 to 0.16 while the minimum attained cavitation number does not change in this region. As mentioned, the amount of ventilation rate to sustain a supercavity is less than the amount of ventilation rate to generate it and Fig.1 shows that by decreasing the ventilation coefficient from 0.16 to 0.06 the clear supercavity with the twin vortex mechanism of gas leakage will sustain. However, it can be seen that the reduction of ventilation coefficient below the 0.06 value will change the mechanism of gas leakage from the twin vortex to the portion gas leakage. The value of ventilation coefficient of 0.06 where the clear supercavity collapses and changes to the portion gas leakage is named the collapse ventilation coefficient or C_{Qc} .

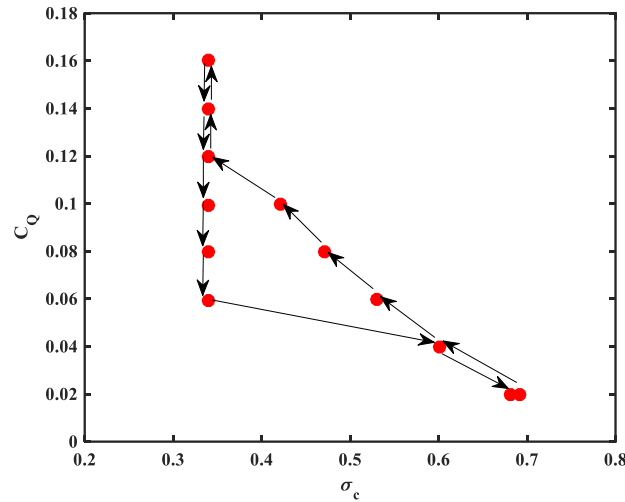


Fig. 1. Experimental results of ventilation coefficient vs. cavitation number. Data is digitized from [9].

Spurk [19] suggested the following model for the portion gas leakage mechanism and showed that measured results are in good agreement with experimental results under high Froude number condition ($Fr > 25$) and for re-entrant jets.

$$C_{Q,Spurk} = k_{Q,Spurk} \frac{(1 + \sigma)}{\sigma} \sqrt{\frac{1}{\sigma} \ln \frac{1}{\sigma}} \quad (7)$$

where $k_{Q,Spurk}$ is an empirical coefficient which is a function of Reynolds number. The empirical functionality of this coefficient to the Reynolds number is not presented in the literature. The only presented value of $k_{Q,Spurk}$ by Spurk is equal to 0.013 at $Re_{D_n} = 0.535 \times 10^6$, where D_n is the cavitator diameter which is used as the characteristic length for determining the Reynolds number [19]. Also, Zou et al. [20] have done a numerical study to investigate the internal velocity distribution and gas loss of supercavities. They have validated their numerical studies by results of experiments for supercavitating flows at $Re_{D_n} = 0.535 \times 10^6$ which are presented by Spurk [19] and have presented the results of the ventilation coefficient vs. cavitation number for different Reynolds numbers. They have also presented the values of $k_{Q,Spurk}$ for seven different Reynolds numbers from 0.224×10^6 to 0.784×10^6 and shown that the gas leakage reduces with increasing the Reynolds number for a given cavitation

number. However, the accuracy of the model was checked only for one value of the Reynolds number and also, the presented range of Reynolds number was not wide. The accuracy of the Spurk formula will be evaluated for the experiments in water tunnels in the following.

Cox and Clayden calculated the gas leakage of the twin vortex mechanism for the first time [21]. Epshtein has also presented a semi-empirical formula which is an extension of the Cox-Clayden model and is more reliable and more widely accepted. The Epshtein formula is presented as follows [22]:

$$C_{Q,Epshtein} = \frac{0.42C_{D0}^2}{\sigma(\sigma^3 Fr^4 - 2.5C_{D0})} \quad Fr \sim 4: 20 \quad (8)$$

where C_{D0} is the drag coefficient of cavitator when the cavitation number is equal to zero. The accuracy of the Epshtein model for the calculation of the formation ventilation rate of the ventilated supercavity in water tunnels will be evaluated in the following.

It should be noted that for experiments conducted in water tunnels, the blockage effect of tunnel walls can affect the shape of the supercavity. The blockage ratio, B , is defined as the ratio of equivalent tunnel diameter (D_T) to cavitator diameter (D_n). Also, for each water tunnel, a blockage cavitation number σ_b is defined which results in the difference between the cavitation numbers in a water tunnel and unbounded flow. Karlikov and Sholomovich have presented the transformation of cavitation numbers between water tunnel and unbounded flows as the following equation [23]:

$$\sigma_\infty = \frac{2\sigma_c^2 - \sigma_b^2}{2\sigma_c} \quad (9)$$

where σ_c is ventilation cavitation number and σ_∞ is the transformed cavitation number in unbounded flow at which the supercavity has the same maximum cross-sectional area as one in the water tunnel. Increasing the blockage ratio will result in lowering the effect of walls and in this state, the blockage cavitation number tends to zero and the unbounded cavitation number σ_∞ approaches the ventilation cavitation number σ_c . Zou et al. [23] showed that the blockage cavitation number is about 0.0018 for blockage ratio of 55, and concluded that the effect of walls can be ignored for such conditions. As discussed earlier, by increasing the ventilation rate, the cavitation number will not change after reaching a minimum value. Yang et al. [16] have also presented a formula based on numerical studies for calculation of this minimum cavitation number and showed that the results are in a very good agreement with experimental data. The minimum cavitation number based on their work is a function of Froude number and blockage ratio as follows [23]:

$$\sigma_m = ((2.5C_{D0})^{1/3} + C_1)Fr^{-4/3} + C_2 \quad (10)$$

where the coefficients C_1 and C_2 are functions of the blockage ratio B and are defined as:

$$C_1 = -1.472B^{-0.3755} \quad (11)$$

$$C_2 = 3.885B^{-1.276} \quad (12)$$

The current study focuses upon presenting an empirical model based on available experimental data, to predict the ventilation demand of artificial supercavities in water tunnels. The proposed ventilation rate formula will be used before reaching the minimum cavitation number for which the magnitude of cavitation number does not change by increasing the ventilation rate and this formula can also be used at the formation point of the supercavity.

The available results of experiments by Ahn et al. [9] which are conducted at CNU and SAFL water tunnels and the experimental results from the work of Kim et al. [24] which is done at CNU water tunnel have been used in the present study in order to develop the empirical model of the ventilated supercavity. The proposed model in the present study is similar to the Spurk model which consists of two different parts. Before presenting the proposed new model, the accuracy of two well-known models of ventilation will be investigated. As mentioned in the literature, the Spurk formula is suitable for high Froude numbers ($Fr > 25$) and can not predict the ventilation coefficient at lower Froude numbers [25]. In order to investigate the accuracy of the Spurk formula at different Froude numbers, the calculated results of this model will be compared with the experimental results. As discussed earlier, $k_{Q,Spurk}$ in Eq. (7) is an empirical coefficient which is a function of Reynolds number. The functionality of this coefficient to the Reynolds number is not mentioned in the literature. Here, a third order polynomial of Reynolds number is considered for $k_{Q,Spurk}$ and the constants are determined using the pattern search optimization process in order to minimize the error between the calculated results of the Spurk model and the experimental data. The Spurk model with the third order polynomial of Reynolds number as its empirical coefficient will be:

$$C_{Q,Spurk} = (a_1 Re^3 + a_2 Re^2 + a_3 Re) \frac{(1+\sigma)}{\sigma} \sqrt{\frac{1}{\sigma} \ln \frac{1}{\sigma}} \quad (13)$$

The pattern search optimization algorithm has been used in order to minimize the root mean square (*rms*) of the percentage error between the Spurk model and experimental results. This algorithm attempts to solve a nonlinear optimization problem with linear or nonlinear constraints and bounds. In this method, a subproblem is formulated by

combining the objective function and constraint functions using the Lagrangian and penalty parameters and each subproblem solution represents an iteration. A subproblem formulation is defined as [26]:

$$\theta(dv, \lambda, s, \rho) = f(dv) - \sum_{i=1}^m \lambda_i s_i \log(s_i - C_i(dv)) + \sum_{i=m+1}^{mt} \lambda_i C_{eq,i}(dv) + \frac{\rho}{2} \sum_{i=m+1}^{mt} C_{eq,i}(dv)^2. \quad (14)$$

Components λ_i in the above equation are nonnegative Lagrange multiplier estimates, elements s_i are nonnegative shifts and ρ is the positive penalty parameter. The pattern search algorithm begins by using an initial penalty and continues by minimizing a sequence of subproblems to the required accuracy and satisfying the feasibility conditions and then, the Lagrangian estimates are updated. Otherwise, the penalty parameter is increased by a penalty factor and these steps are repeated until the stopping criteria are met.

In the pattern search method, only the function values have been used for solving the problem instead of using the derivatives or approximation of derivatives. For example, if a function $f(dv)$ is to be minimized where design variables $dv \in R^n$ and the function $f: R^n \rightarrow R$, the general pattern search algorithm can be represented as Fig. 2.

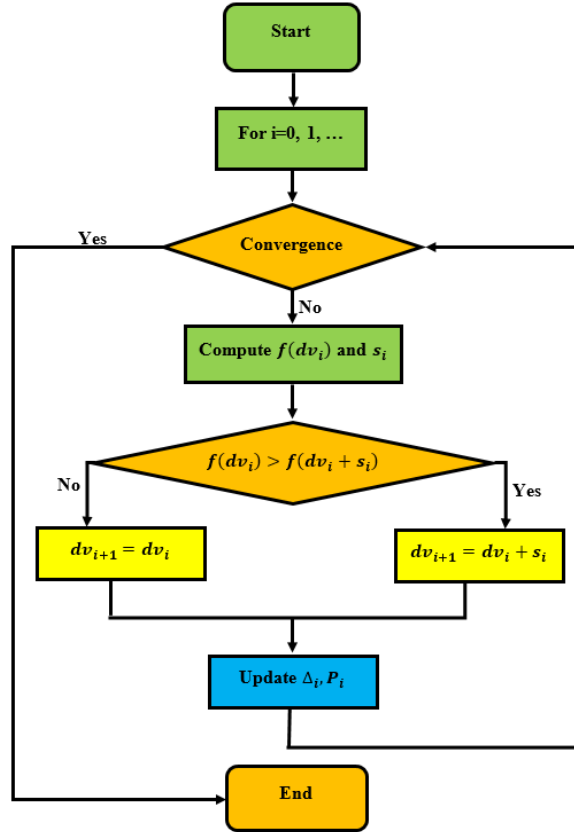


Fig. 2. The flowchart of the pattern search optimization algorithm

As Fig. 2 shows, Δ_i and P_i are the step and the pattern respectively. As mentioned, in the pattern search method, only the simple decrease of the objective function is required [27]. The step s_i is calculated using exploratory moves (Δ_i, P_i) . More information about the pattern search method and also the theory of convergence of this process can be found in [28].

Based on Eq. (13), the coefficients a_1 , a_2 and a_3 are considered as design variables that are unknown and the proper values of these variables are obtained to reach the optimum of the objective function (minimizing the error between the Spurk model and experimental results). The calculated values of these coefficients are: $a_1 = -0.2982$, $a_2 = 0.0853$ and $a_3 = 0.0117$.

The minimum obtained root mean square (*rms*) of the percentage of the error between the calculated results of the Spurk model and the experimental data is approximately equal to 67 %.

As mentioned earlier, the Epshtein formula has been presented in order to model the gas leakage rate of the twin vortex mechanism of ventilated supercavities. In order to evaluate the accuracy of this model for prediction of the ventilation rate of supercavities in the water tunnels, the formation ventilation coefficient will be determined using this model and will be compared to the formation ventilation rates from experimental tests at different Froude numbers. The experimental data of the formation ventilation coefficient are from the SAFL water tunnel at the blockage ratio of 7.15 and at different Froude numbers [25]. The comparison between these experimental data and

the formation ventilation coefficient calculated using the Epshtein formula (Eq. (8)) are depicted in Fig. 3. The results of Fig.3 indicate that the Epshtein model cannot predict the ventilation rate of the twin vortex mechanism at water tunnel tests properly and the *rms* of the percentage of the error between the calculated results of the Epshtein model and the experimental data is approximately equal to 82 %. Also, as Fig. 3 shows, the predicted trend of change of the formation ventilation coefficient using the Epshtein model is different from the experimental results. The reason for such a difference between the results of Epshtein formula and the experimental data is that the experiments for obtaining the Epshtein formula were done at a free surface rotating pool without the effects of blockage [25]. Also, there is a semi-empirical model presented by Campbell and Hilborne for twin vortex gas leakage mechanism which is also based on the experiments at a free surface facility and is not suitable for closed water tunnels [25]. Therefore, there is a lack of an appropriate model to predict the ventilation demand in water tunnel tests with the effect of blockage of walls.

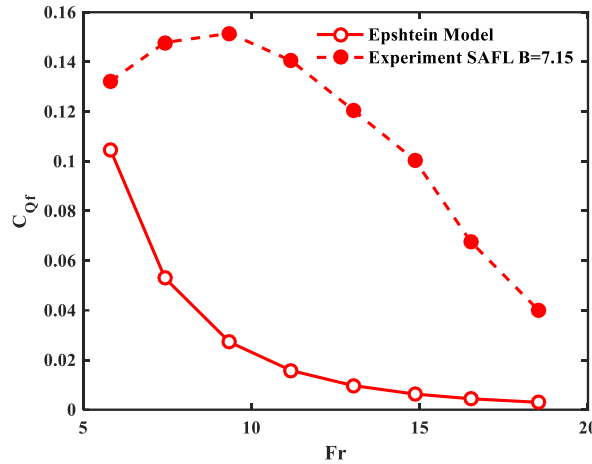


Fig. 3. Comparison between the results of the Epshtein model and the experimental data of the formation ventilation coefficient from the SAFL water tunnel at $B=7.15$ and different Froude numbers [15]

The purpose of the present study is to develop a model for the ventilation coefficient which will be used at both low and high Froude numbers and for water tunnels with the blockage effect. Just like the Spurk formula, the first part of the present model is similar to $k_{Q,Spurk}$ which is considered as a function of Reynolds and Froude numbers (Re and Fr) in order to consider the effect of the both of these dimensionless numbers in calculations and the second part of the formula is a function of unbounded cavitation number (σ_∞). The second part of the formula which is a function of the unbounded cavitation number is considered to be a second-order polynomial. The proposed empirical model is considered as follows:

$$C_{Q,new Model} = f_1(M)f_2(\sigma_\infty) \quad (17)$$

$$f_1(M) = aM^3 + bM^2 + cM$$

$$M = \frac{(ReFr^2)^{\frac{1}{3}}}{100}$$

$$f_2(\sigma_\infty) = d\sigma_\infty^2 + e\sigma_\infty + f.$$

In the above equation, M is a nondimensional parameter which is a function of Re and Fr and $f_1(M)$ is a third-order polynomial. Empirical coefficients a, b, c, d, e and f are considered as design variables which are unknown and the proper values of these variables are calculated using the pattern search optimization algorithm to minimize the objective function which is considered as the *rms* of the percentage of error between the calculated ventilation rates of the proposed model and the experimental data.

The calculated values of these coefficients using the pattern search method are $a = 0.0005$, $b = -0.00498$, $c = 0.0131$, $d = 51.3683$, $e = -66.0229$, $f = 23.1161$. The *rms* of the percentage of error between the calculated results of the proposed model and the experimental data is approximately equal to 29 % which is relatively smaller than that of the Spurk model (67 %). The normal probability density function (pdf) is defined as:

$$f(x, \sigma, \mu) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \quad -\infty < x < \infty \quad (18)$$

which the first parameter μ is the mean of the normal distribution which is a scalar value and the second parameter σ is the standard deviation of the normal distribution and is a nonnegative scalar value. Parameters related to the normal distribution of the proposed model and the Spurk model are presented in Table 1. As results show, the mean and the standard deviation related to the results of the proposed model are approximately equal to -14 % and 25.8 % respectively. However, the values of the mean and the standard deviation related to the Spurk model are approximately equal to -54 % and 39.8 % respectively. As can be seen, the mean of the proposed model is relatively closer to zero in comparison with the results of the Spurk model. Also, the standard deviation related to the proposed model is approximately 35 % lower compared to the Spurk model. Therefore, the results indicate that the new proposed model can predict the ventilation coefficient of ventilated supercavities in water tunnels with acceptable accuracy.

Table1 The parameters of the normal distribution for the proposed model and the Spurk model

Parameter	New model	Spurk model
Mean (μ)	-14.01 %	-54.11 %
Standard deviation (σ)	25.78 %	39.79 %

The summary of the method for the proposed model can be observed in Fig. 4

The comparison between the results of the proposed model and the experimental data for the calculation of the ventilation coefficient of ventilated supercavities in water tunnels before the formation point and for the formation ventilation coefficient will be presented in the following section.

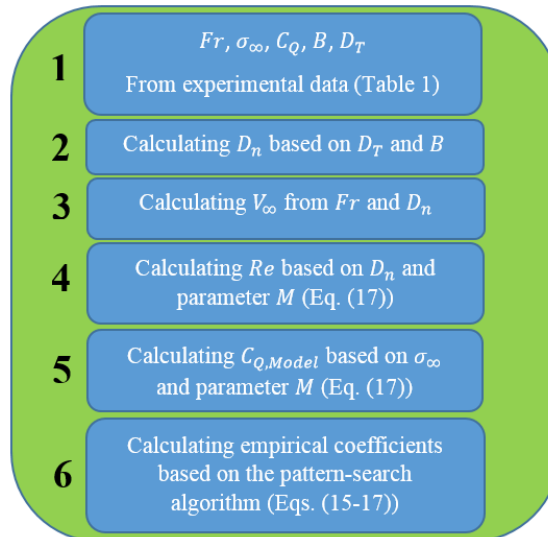


Fig. 4. Summary of the method for the proposed model

Results and discussion:

The calculated ventilation coefficients using Eq. (17) with determined coefficients using the pattern search algorithm are compared with the experimental data are shown in Figs. 5-6. As mentioned in the previous section, the value of rms of the percentage of error between the model results and the experimental data from the work of Ahn et al. [9] and Kim et al. [24] is minimized using the pattern search optimization algorithm and is approximately equal to 29 %. The empirical coefficients in Eq. (17) which are determined in the optimization process are presented in the previous section. Therefore, it can be seen that the proposed model can predict the ventilation coefficient of ventilated supercavities in water tunnels with an acceptable accuracy.

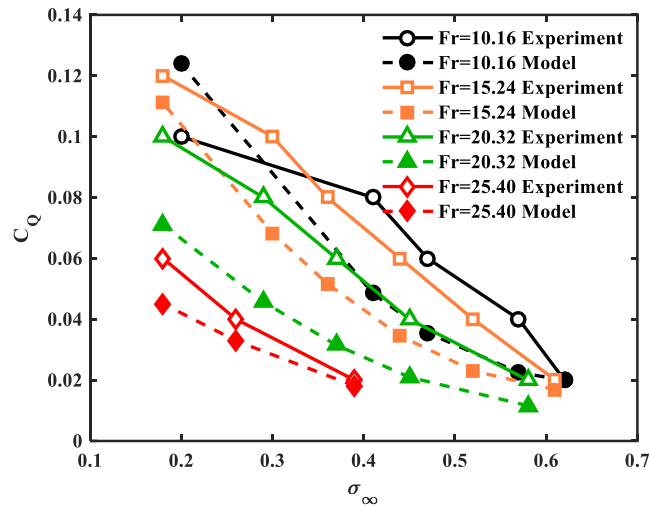


Fig. 5. Comparison between the experimental data of CNU water tunnel [9] at different Froude numbers and $B=7.15$ and results of the proposed model

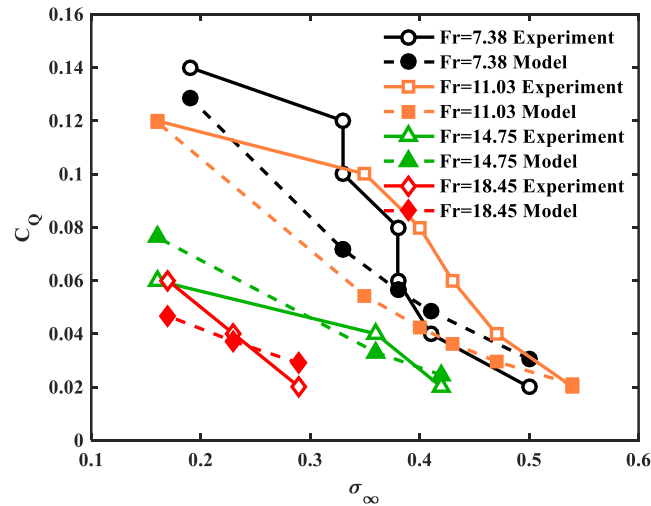


Fig. 6. Comparison between the experimental data of SAFL water tunnel [9] at different Froude numbers and $B=7.15$ and results of the proposed model

In order to investigate the precision of the presented model for the calculation of the formation ventilation rate of supercavity, the experimental data of the formation ventilation rate from the work of Shao et al. [25] are used at different Froude numbers and are depicted in Figs. 7-8 in comparison to the calculated values of the proposed model (Eq. (17)). It should be noted that for the calculation of the formation ventilation coefficient at different Froude numbers, the unbounded cavitation number in Eq. (17) is set to the minimum cavitation number (Eq. (10)) which is then transferred to the unbounded minimum cavitation number using Eq. (9). As can be seen in Figs. 12-13, the presented model can predict the formation ventilation coefficient with acceptable accuracy. It is shown in Table 2 that the maximum percentage of error between the calculated values of the model and the experimental data is equal to 28.8 %. Also, the root mean square of the percentage of error between the model and the experimental results is equal to 19.3 %. Furthermore, it can be seen that the proposed model can predict the trend of variations of the formation ventilation coefficient vs. Froude number. Figs. 7-8 show that for different water tunnel facilities, the formation ventilation coefficient C_{Qf} increases with Froude number increase (or flow velocity increase), reaches a peak at a critical Fr and then decreases which is consistent with the results from [25] and the model can approximately predict the same trend of variations. Therefore, the proposed model can estimate the required ventilation rate for the formation of the supercavity with acceptable accuracy and also can predict the trend of variation of C_{Qf} just like the water tunnel tests (except for the case of Fig. 7 with a little different trend). Fig. 7 and Fig. 8 show the variations of formation ventilation coefficient vs. Froude number at two different water tunnels (CNU and SAFL respectively) with the same blockage ratio ($B = 7.15$). The trends of variations of C_{Qf} vs. Froude number is similar for two facilities but in particular, the SAFL tests show a larger peak value of C_{Qf} in comparison



to that of CNU water tunnel. Also, it can be seen that the critical Froude number is lower for the SAFL water tunnel as compared to the CNU water tunnel. Shao et al. [25] have discussed that this leftward and upward shift of the C_{Qf} in SAFL water tunnel compared to the CNU water tunnel is because of the mismatch of Reynolds numbers between the experiments of two tunnels. In fact, to keep both experiments at the same blockage ratio (equal to 7.15) and because of the difference between the test section sizes of these two water tunnels, the cavitator sizes were different which resulted in different Reynolds numbers at the same Froude numbers. The proposed model in this study considers both the Reynolds and Froude numbers and thus can predict the variations of the formation ventilation coefficient with an acceptable accuracy.

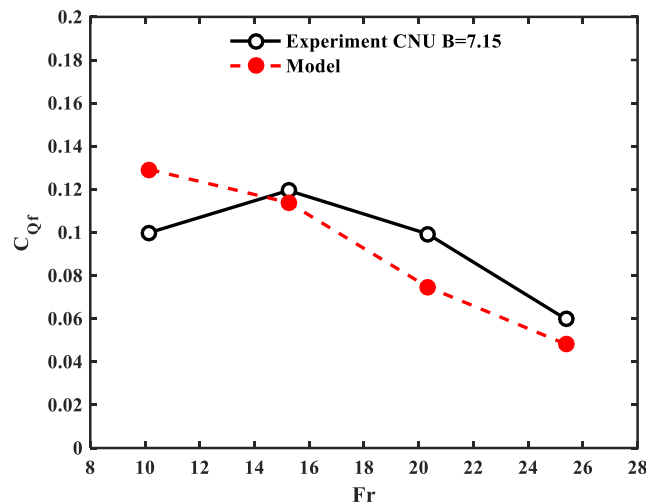


Fig. 7. Comparison between the experimental data of formation ventilation rate in CNU water tunnel at different Froude numbers and $B=7.15$ [25] and results of the presented model

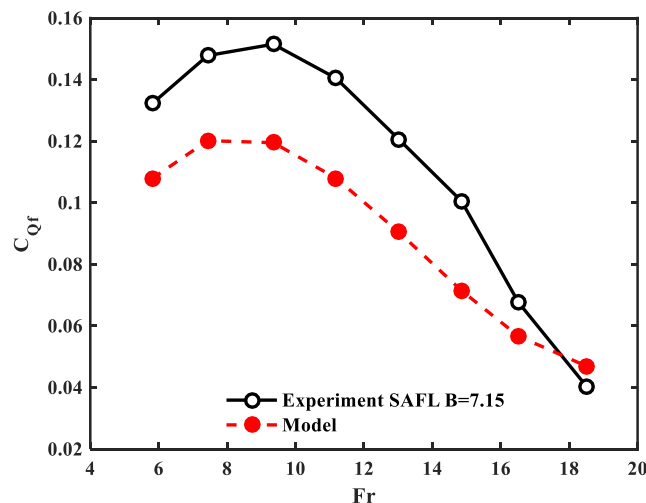


Fig. 8. Comparison between the experimental data of formation ventilation rate in SAFL water tunnel at different Froude numbers and $B=7.15$ [25] and results of the presented model

Table 2 Comparison between the experimental data of two water tunnels form [25] and the results of the presented model

Data #	Tunnel	Fr	Blockage (B)	C_{Qf} experiment	C_{Qf} model	% error
1	CNU	10.16	7.15	0.1000	0.1292	29.2
2	CNU	15.24	7.15	0.1200	0.1139	5.1
3	CNU	20.32	7.15	0.1000	0.0748	25.2
4	CNU	25.4	7.15	0.0600	0.0481	19.8
5	SAFL	5.81	7.15	0.1322	0.1079	18.4

6	SAFL	7.43	7.15	0.1478	0.1202	18.7
7	SAFL	9.35	7.15	0.1515	0.1195	21.1
8	SAFL	11.19	7.15	0.1404	0.1080	23.1
9	SAFL	13.02	7.15	0.1206	0.0905	25.0
10	SAFL	14.85	7.15	0.1005	0.0714	29.0
11	SAFL	16.52	7.15	0.0676	0.0567	16.1
12	SAFL	18.52	7.15	0.0401	0.0468	16.7
13	CNU	8.85	5.36	0.1000	0.0963	3.7
14	CNU	13.15	5.36	0.1000	0.0859	14.1
15	CNU	17.54	5.36	0.0800	0.0568	29.0
16	CNU	22.08	5.36	0.0400	0.0363	9.3
17	SAFL	6.92	10.72	0.1142	0.1398	22.4
18	SAFL	9.23	10.72	0.1390	0.1558	12.1
19	SAFL	11.46	10.72	0.1554	0.1528	1.7
20	SAFL	13.85	10.72	0.1497	0.1358	9.3
21	SAFL	15.88	10.72	0.1469	0.1150	21.7
22	SAFL	18.19	10.72	0.1184	0.0901	23.9
23	SAFL	20.46	10.72	0.0907	0.0695	23.4
24	SAFL	22.85	10.72	0.0612	0.0585	4.4
					<i>max(% error)</i>	29.2
					<i>rms(% error)</i>	19.4

Conclusion

In summary, our study presents an empirical model for the formation ventilation coefficient within closed-wall water tunnels, specifically addressing the influence of wall effects on supercavity dynamics. Validated across a broad range of Froude numbers (from 5 to 32), this model effectively predicts the relationship between the ventilation coefficient and cavitation number for various tunnel configurations. By leveraging a pattern search optimization algorithm, we determine empirical constants that minimize the percentage of error between the model's predictions and experimental data. Notably, our proposed model accurately estimates the required ventilation rate for supercavity formation and maintenance, serving as a valuable predictive tool for enhancing marine engineering. With an acceptable level of accuracy, it enables precise control over ventilation requirements in underwater vehicles, aligning well with experimental results across different water tunnels and blockage ratios.

References

- [1] A. Karn and B. Rosiejka, 'Air entrainment characteristics of artificial supercavities for free and constrained closure models', *Exp. Therm. Fluid Sci.*, vol. 81, pp. 364–369, 2017, doi: 10.1016/j.expthermflusci.2016.10.003.
- [2] M. Mirzaei, M. M. Alishahi, and M. Eghtesad, 'High-speed underwater projectiles modeling: a new empirical approach', *J. Brazilian Soc. Mech. Sci. Eng.*, vol. 37, no. 2, pp. 613–626, 2015, doi: 10.1007/s40430-014-0190-7.
- [3] Balint Vanek, 'Control methods for high-speed supercavitating vehicles', University of Minnesota, 2008.
- [4] E. Kawakami and R. E. A. Arndt, 'Investigation of the Behavior of Ventilated Supercavities', *J. Fluids Eng.*, vol. 133, no. 9, pp. 091305 (1–11), 2011, doi: 10.1115/1.4004911.
- [5] R. Shafaghat, S. M. Hosseinalipour, I. Lashgari, and A. Vahedgermi, 'Shape optimization of axisymmetric cavitators in supercavitating flows, using the NSGA II algorithm', *Appl. Ocean Res.*, vol. 33, no. 3, pp. 193–198, 2011, doi: 10.1016/j.apor.2011.03.001.
- [6] J. K. Choi, B. K. Ahn, and H. T. Kim, 'A numerical and experimental study on the drag of a cavitating underwater vehicle in cavitation tunnel', *Int. J. Nav. Archit. Ocean Eng.*, vol. 7, no. 5, pp. 888–905, 2015, doi: 10.1515/ijnaoe-2015-0062.
- [7] H. Mokhtarzadeh, 'Supercavitating vehicle modeling and dynamics for control', University of Minnesota, 2010.
- [8] J. H. Guo, C. J. Lu, Y. Chen, and J. Y. Cao, 'Study of ventilated cavity morphology in different gas leakage regime', *J. Hydrodyn.*, vol. 22, no. 5 SUPPL. 1, pp. 778–784, 2010, doi: 10.1016/S1001-6058(10)60030-3.
- [9] B. K. Ahn, S. W. Jeong, J. H. Kim, S. Shao, J. Hong, and R. E. A. Arndt, 'An experimental investigation of artificial supercavitation generated by air injection behind disk-shaped cavitators', *Int. J. Nav. Archit. Ocean Eng.*, vol. 9, no. 2, pp. 227–237, 2017, doi: 10.1016/j.ijnaoe.2016.10.006.
- [10] B. Ji, X. W. Luo, X. X. Peng, Y. Zhang, Y. L. Wu, and H. Y. Xu, 'Numerical investigation of the ventilated cavitating flow around an under-water vehicle based on a three-component cavitation model', *J. Hydrodyn.*, vol. 22, no. 6, pp. 753–759, 2010, doi: 10.1016/S1001-6058(09)60113-X.

- [11] R. L. Waid, 'Water tunnel investigation of two-dimensional cavities', Pasadena, California, 1957.
- [12] L. G. Straub, M. W. Self, J. F. Ripken, and O. Of, 'Steady-state cavity studies in a free-jet water tunnel', University of Minnesota, 1955.
- [13] V. N. Semenenko, 'Artificial Supercavitation: physics and calculation', *Lecture notes from the RTO AVT/VKI special course on supercavitating flows*. von Karman Institute for fluid dynamics, Rhode Saint Gene'se, Belgium, pp. 11.1-11.33, 2001.
- [14] A. Karn, R. E. A. Arndt, and J. Hong, 'Dependence of supercavity closure upon flow unsteadiness', *Exp. Therm. Fluid Sci.*, vol. 68, pp. 493–498, 2015, doi: 10.1016/j.expthermflusci.2015.06.011.
- [15] Z. Wang, B. Huang, G. Wang, M. Zhang, and F. Wang, 'Experimental and numerical investigation of ventilated cavitating flow with special emphasis on gas leakage behavior and re-entrant jet dynamics', *Ocean Eng.*, vol. 108, pp. 191–201, 2015, doi: 10.1016/j.oceaneng.2015.07.063.
- [16] W. Yang, Z. Yang, and K. Wen, 'Numerical investigation on the gas entrainment rate on ventilated supercavity body', *J. Comput. Multiph. Flows*, vol. 8, no. 4, pp. 169–177, 2016, doi: 10.1177/1757482X16654021.
- [17] C. Brennen, 'A numerical solution of axisymmetric cavity flows', *J. Fluid Mech.*, vol. 37, no. 4, pp. 671–688, 1969, doi: 10.1017/S0022112069000802.
- [18] A. Karn, R. E. A. Arndt, and J. Hong, 'An experimental investigation into supercavity closure mechanisms', pp. 259–284, 2016, doi: 10.1017/jfm.2015.680.
- [19] J. H. Spurk and B. Knig, 'On the gas loss from ventilated supercavities', *Acta Mech.*, no. 3–4, pp. 125–135, 2002, doi: 10.1007/BF01176238.
- [20] W. Zou, L.-P. Xue, W.-W. Jin, and X.-T. Xiang, 'Investigation of internal flow velocity distribution and gas loss of high-speed supercavitating flows', in *Proceedings of the ASME 2017 Fluids Engineering Division Summer Meeting FEDSM2017*, 2017, doi: 10.1115/FEDSM2017-69468.
- [21] R. N. Cox and W. A. Clayden, 'Air entrainment at the rear of a steady cavity', in *Proceedings of the Symposium on Cavitation in Hydrodynamics*, 1955.
- [22] L. A. Epshtein, *Methods of theory of dimensionality and similarity in problems of ship hydromechanics*. Sudostroenie Publishing House, Leningrad, 1970.
- [23] Z. O. U. Wang, Y. U. Kaiping, R. E. A. Arndt, and E. Kawakami, 'On minimum cavitation number of the ventilated supercavity in water tunnel', vol. 56, no. 10, pp. 1945–1951, 2013, doi: 10.1007/s11433-012-4917-0.
- [24] K. Byeung-jin, C. Jung-kyu, and K. Hyoun-ae, 'An experimental study on ventilated supercavitation of the disk cavitator', *J. Soc. Nav. Archit. Korea*, vol. 52, no. 3, pp. 236–247, 2015.
- [25] S. Shao, A. Karn, B. K. Ahn, R. E. A. A. Arndt, and J. Hong, 'A comparative study of natural and ventilated supercavitation across two closed-wall water tunnel facilities', *Exp. Therm. Fluid Sci.*, vol. 88, pp. 519–529, 2017, doi: 10.1016/j.expthermflusci.2017.07.005.
- [26] T. G. Kolda, R. M. Lewis, and V. Torczon, 'A generating set direct search augmented Lagrangian algorithm for optimization with a combination of general and linear constraints', Tech. Report SAND2006–5315, Albuquerque, New Mexico and Livermore, California Sandia National Laboratories, 2006.
- [27] V. Torczon, 'Pattern Search Methods for Nonlinear Optimization', pp. 6–8, 1995.
- [28] V. J. Torczon, 'On the Convergence of Pattern Search Algorithms', *SIAM J. Optim.*, vol. 7, no. 1, pp. 1–25, 1997, doi: 10.1137/S1052623493250780.